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**Euclid and Mathematical Theorems**

Euclid was a mathematician famous for his collection of 13 books that deal with on geometry. What makes his books so distinctive are that his books are the most read apart from the bible. Euclid has been referred to as the father of geometry because of his outstanding : he contribution. He was born around 365 B.C, and raised in Alexandria, Egypt, during the reign of Ptolemy. He has attended Plato Academy in Athens for some time. Euclid's textbook, the "Stoicheion" or "Elements," became known as the most successful mathematical textbook of all time (Grattan-Guinness, 355). The Elements is a clear and comprehensive compilation of all known mathematics of the time, which offered a great explanation of the works of Pythagoras, Hippocrates, Theudius, Theaetetus, and Exodus.

] I want to see you reference Heath for Euclid Biography.

The total concepts enumerated in the book were 465 theorems and proofs expounded by the use of a compass and a straight edge. To establish proven theorems, the book Elements, models for the mathematical argument followed logical deductions from initials deductions which Euclid classified as axioms and postulates (Heath).

The five general axioms are as follows:

What's the difference?

1. Things which are equal to the same thing are equal to each other
2. If equals are added to equals the wholes are equal.
3. If equal is subtracted from the equation, the difference is equal.
4. Things that coincide with one another are equal.
5. The whole is greater than apart.

The geometrical postulates are as follows:

1. It is possible to draw a line from any point to any point.
2. A finite line can be extended continuously in a straight line.
3. With any center and distance, it is possible to create a circle.
4. Right angles are equal to each other and finally.
5. If a straight line crossing two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

The volume of elements also contained mathematical formulas for calculating the volumes of solids like cones and pyramids. It also offers proof about geometric series perfect and prime numbers and the algorithms for computing Greatest common divisor, GCD, and the least common multiple LCM algorithms. Elements also offered a series of theorems on numbers and integer properties, which informed the early conceptualization of the number theory. The Elements is a collection of 13 books, with each set of books dealing with a specific area of mathematics. Books 1-6 deals with the plane geometry, 7-9 deals with the number theory, book ten deals with the Exodus theory of irrational numbers, and 11 to 13 deals with solid geometry. As advanced earlier, Euclid's discoveries were few and what he mostly did was base his work on those who had contributed to maths before him. Most of the proofs for those earlier works were his, and he added value to his predecessors. From his work, Euclidian geometry arose, which is the study of points, lines, planes, and other geometric figures.

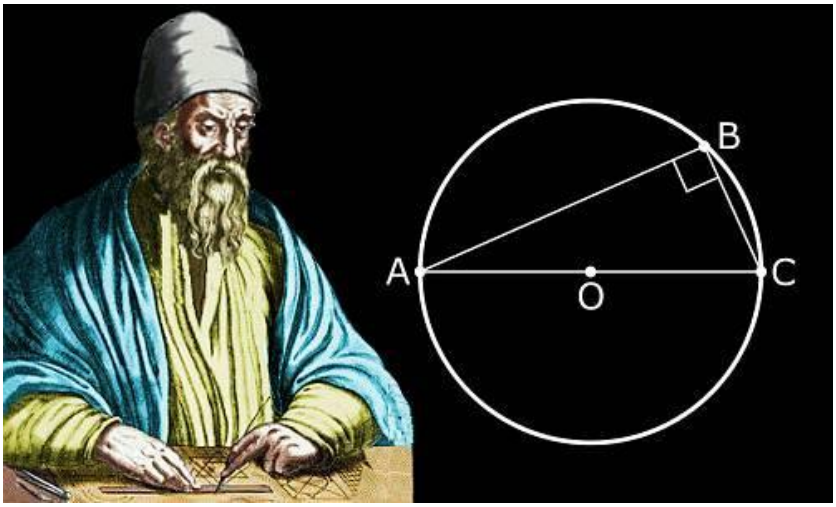



*Figure 1: Euclid is widely regarded as the father of Geometry.*

**Euclid Theorems**

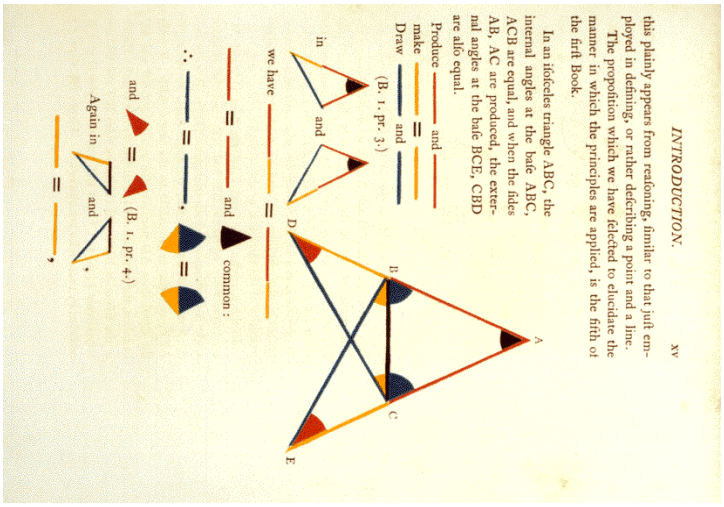
Euclid contributed to the development of many theorems through his postulates. He did not only state the postulates but went a bit further to offer the proof for the statements (Janičić, Predrag, and Kordić, 723). His theorems include "pons asinorum," derived from proposition 5 of book one and is commonly known as the Isosceles triangle theorem. The sum of the angles theorem is a significant theorem that Euclid advanced.

**The Isosceles Triangle theorem**

The theorem states that angles opposite the equal sides of an isosceles triangle are equal themself. This statement also means that if two angles of a triangle are equal, then the two sides are also equal (Eberl). An isosceles triangle has two sides equal in length. It has three interior angles and three acute angles. To prove that opposite sides and angles are the same, a bisector is introduced on the base, making the two triangles share a common side; applying the SAS congruence criterion, the two angles are equal. This theory has been developed over the years, while still maintaining the original statements that Euclid made.

This doesn't need to be explained.

Math arguments should be distinct



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this plainly appears from reafoning, fimilar to that juft employed in defining, or rather deferibing a point and a line.

The propofition which we have felected to elucidate the manner in which the principles are applied, is the fifth of the firft Book.

In an ifofceles triangle ABC, the internal angles at the bafe ABC, ACB are equal, and when the fides AB, AC are produced, the external angles at the bafe BCE, CBD are alfo equal.

Produce ——— and ———
make ——— = ———
Draw ——— = ——— and ———
(B. 1, pr. 3.)

in ——— and ———
we have ——— = ——— and ——— common :
∴ ——— = ——— and ———
and ▲ = ▲ (B. 1, pr. 4.)

Again in ——— and ———
——— = ———,

*Figure 2: An early edition of Elements showing part of proof for the isosceles theorem.*

Euclid added a second conclusion to the isosceles theorem. He observed that if equal sides of the triangle extend below the base, then the angles between the extensions and the base are also equal. Although Euclid never used this conclusion, proof can be found by drawing auxiliary lines to the sides of the triangle. Questions arise over why Euclid opted to include the second conclusion, given that it made proof more difficult. Later mathematicians thought that the second conclusion was necessary to ensure that other propositions which Euclid did not cover were tested under it and rejected if there was no proof. This second proof today relies upon the Side-Angle-Side, also a postulate by Euclid.

**Sum of the Angles of a Triangle Theorem**

This theorem holds that the sum of interior angles of a triangle equals 180°. While Euclid did not use the words stated prior, his triangle postulate posited that the sum of angles of a triangle is equal to two right angles. In his first book, Euclid states “In any triangle, if one of the sides is created then the exterior angle equals the sum of the two interior and opposite angles and the sum of the three interior angles of the triangle equals two right angles” (Bryne). As seen in

figure 3, the initial triangle is ABC. We extend line B.C. to point D. We then draw a line C.E. which is parallel to line A.B. Because A.C. and C.E. are parallel alternate angles ACE and BAC are equal to each other. The exterior angle ECD equal the interior and opposite angle ABC because A.B. and E.C. are parallel and are on the same straight line B.D. ACE was earlier proven equal to BAC; thus, angle ACD is equal to the sum of BAC and ABC's two interior and opposite angles.

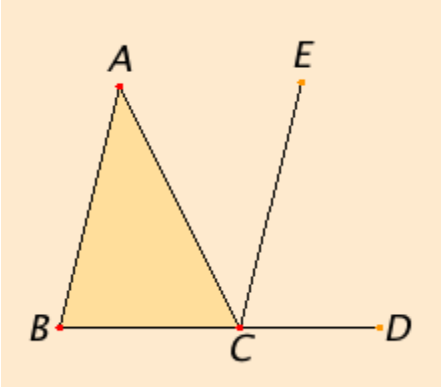


*Figure 3: Sum of the angles proof*

The sum of the angles ACD and ACB equals the sum of ABC, BCA, and CAB. Keeping in mind that the sum of angles ACD and ABC are equal to two right angles is equal to the sum of ABC, BCA, and CAB. Since the latter statement is valid, the theorem as postulated by Euclid that the sum of interior angles in a triangle is equal to two right angles has been proven true. The parallel line theorem holds a central place in the proof of the sum of angles of a triangle theorem.

All the theorems that Euclid participated in writing were all related to maths. Though not famous because of the originality of the ideas, Euclid combined all his predecessors' maths knowledge and put them together in books. He is credited for combining all the mathematicians and coming up with a book that was easy to understand. Making it easier to impart knowledge to other scholars who came after him. His explanations were also straightforward, thus simplifying

the perceived complex subject for understanding by many. Some of his postulates can be applied in physics subjects and are not just limited to mathematics.

In geometry, where Euclid is considered the founder, there have been many improvements, although most Euclid postulates still hold up to date. Before the 19th century, geometry, one of the oldest branches of mathematics, was exclusively devoted to Euclidean geometry. The Euclidean geometry includes the line, point, plane, distance, angle, surface, and curve, forming its basis (Krause). However, discoveries during the 19th century expanded the scope of geometry. Geometry now boasts of several other sub-branches and influences mathematics, having its application in almost all sciences, including architecture, graphics, and arts.

Another development in the field of geometry is the rise of non-Euclidian geometry. In other words, its geometry is not covered by the postulates of Euclid. It mainly explains hyperbolic geometry, with hyperbolic and spherical geometries different from Euclidian geometry, although they are closely related. The non-Euclidian geometry developed along two historical threads; one was the movement of the stars and planets in the seemingly hemispherical sky. The second manner in which this geometry arose was to prove Euclid's five postulates about parallel as a theorem. The influence of Euclid in the development of geometry is evident because most of his postulates form the basics of the subject through which further progress was able to be made.

@ INCLUDE a picture for hyperbolic, spherical and flat geometries.

Check grammar and punctuation.

Include a bit more about the elements, maybe details about Euclid's original contributions and list theorems that [illegible] with acutenes. [illegible]. Light

**Work Cited**

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